

Math 581 Homework 5

April 21, 2022

Read the section on connectedness in chapter 4.

Problem 1 (Problem 3-21).

Problem 2 (Problem 3-22).

Problem 3 (Problem 3-23).

Problem 4 (Problem 3-24).

Problem 5. Prove that \mathbb{Z}_p^\wedge is **totally disconnected**: the connected components are exactly the singletons.

Problem 6 (Problem 4-13).

Problem 7 (Problem 4-14).

Problem 8 (Problem 4-15).

Problem 9. Let $\{X_\alpha\}_{\alpha \in A}$ be arbitrary family of topological spaces. Let $X = \prod_\alpha X_\alpha$ (with the product topology).

(i) Prove that if each X_α is path-connected, then X is path-connected.

(ii) Prove that if each X_α is connected, then X is connected. (Hint: Use that you know this is true for finite products. But be careful... There are several ways to prove this, and many ways to not prove this.)

Problem 10. (a) Describe the connected components of $\mathrm{GL}_n(\mathbb{R})$.

(b) Prove that $\mathrm{GL}_n(\mathbb{C})$ is connected.