Math 581 Homework 2

April 13, 2022

Read Chapter 2 of Lee's *Introduction to Topological Manifolds*.

Problem 1 (Problem 2-5)**.**

Problem 2 (Problem 2-10)**.**

Problem 3 (Problem 2-12)**.**

Problem 4 (Problem 2-20)**.**

Problem 5 (Problem 2-25)**.**

Problem 6. Let X be a topological space and $\mathcal{B}$ a subbasis for the topology on X. Prove that the following statements are equivalent:

(1) For every collection $\mathcal{U}$ of open subsets of X such that $\bigcup \mathcal{U} = X$, there is a finite subcollection $U_1, ..., U_n$ so that $X = U_1 \cup \cdots \cup U_n$.

(2) For every collection $\mathcal{U} \subseteq \mathcal{B}$ such that $\bigcup \mathcal{U} = X$, there is a finite subcollection $U_1, ..., U_n$ so that $X = U_1 \cup \cdots \cup U_n$.

Hint: For (2) $\Rightarrow$ (1), argue by contradiction, assuming $\mathcal{U}$ is a maximal counterexample to the statement in (1). Find a point outside of $\bigcup \mathcal{B} \cap \mathcal{U}$ and append various subbasis neighborhoods of that point to $\mathcal{U}$; extract consequences from maximality and try to get a contradiction.