Math 303 Homework 4

Exercise 1. Express the following sets in the indicated form of notation.

(a) $\{n \in \mathbb{Z} \mid n^2 < 20\}$ in list notation;

(b) $\{4k + 3 \mid k \in \mathbb{N}\}$ in implied list notation;

(c) The set of all odd multiples of six in set-builder notation;

(d) The set $\{1, 2, 5, 10, 17, \ldots, n^2 + 1, \ldots\}$ in set-builder notation.

Exercise 2. Find sets X_n for each $n \in \mathbb{N}$ such that $X_{n+1} \subsetneqq X_n$ for all $n \in \mathbb{N}$. Can any of the sets X_n be empty?

Exercise 3. Express the set $\mathcal{P}(\{\varnothing, \{\varnothing, \{\varnothing\}\}\})$ in list notation.

Exercise 4. Let X be a set and let $U, V \subseteq X$. Prove that U and V are disjoint if and only if $U \subseteq X \backslash V$.

Exercise 5. For each of the following statements, determine whether or not it is true for all sets A and X, and prove your claim.

(a) If $X \backslash A = \varnothing$, then $X = A$.

(b) If $X \backslash A = X$, then $A = \varnothing$.

(c) If $X \backslash A = A$, then $A = \varnothing$.

(d) $X \backslash (X \backslash A) = A$.

Exercise 6. For each of the following statements, determine whether it is true for all sets X, Y, false for all sets X, Y, or true for some choices of X and Y and false for others.

(a) $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$

(b) $\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$

(c) $\mathcal{P}(X \times Y) = \mathcal{P}(X) \times \mathcal{P}(Y)$

(d) $\mathcal{P}(X \backslash Y) = \mathcal{P}(X) \backslash \mathcal{P}(Y)$

Exercise 7. Let F be a set whose elements are all sets. Prove that if $\forall A \in F,\ \forall x \in A,\ x \in F$, then $F \subseteq \mathcal{P}(F)$.