

Math 303 Homework 12

November 13, 2022

Exercise 1. Suppose $a \equiv b \pmod{m}$ and $n \geq 0$ is an integer. Prove that $a^n \equiv b^n \pmod{m}$.

Exercise 2. Prove that

$$1^n + 2^n + \cdots + (n-1)^n$$

is divisible by n for any odd integer n .

Exercise 3. Prove that there are no integers x and y such that

$$15x^2 - 7y^2 = 9$$

Exercise 4. Show that $7^{120} - 1$ is divisible by 143. (If you find yourself dealing with large numbers, then you are taking the long way. See if you can apply a theorem from the notes...)

Exercise 5. Let p be a prime. Prove that $(p-1)! \equiv -1 \pmod{p}$.