Math 303 Homework 11

November 7, 2022

Exercise 1. Consider the prime numbers $2 = p_1, 3 = p_2, p_3, p_4, \ldots$. What is the smallest number n for which

$$p_1 \cdots p_n + 1$$

is not prime? (This isn't a 'proof' exercise, just compute until you get one that isn't. I promise $n < 10$ so it won't be too bad.)

Exercise 2. Let a and b be nonzero integers. If there exist integers r and s with $ar + bs = 1$, show that a and b are coprime (i.e. show that $\gcd(a, b) = 1$). Hint: The only nonnegative divisor of 1 is 1 itself.

Exercise 3. Let $(F_n)_{n \geqslant 0}$ be the Fibonacci sequence, where $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$. Prove that F_n and F_{n+1} are coprime.

Exercise 4. Let $x, y \in \mathbb{Z}_{\geqslant 0}$ be coprime. If xy is a perfect square (i.e. the square of an integer) prove that both x and y are perfect squares. Hint: Use the Fundamental Theorem of Arithmetic.

Exercise 5. Prove that there are an infinite number of primes of the form $6n + 5$.