Math 303 Homework 10

November 1, 2022

Exercise 1. Prove that if $a|b$ and $b|c$ then $a|c$. (See Definition 6.1.4 for this notion.)

Exercise 2. Prove that if $a|b$ and $a|c$ then $a|(bx + cy)$ for any integers x and y.

Exercise 3. Use the Euclidean algorithm to compute the greatest common divisor of 7125 and 1300. Show all of your steps.

Exercise 4. Use the reverse Euclidean algorithm (see Example 6.1.33 and the text before and after) to find integers x and y such that

$$630x + 385y = 4340.$$

Exercise 5. Compute the gcd of various pairs of Fibonacci numbers (remember, those are the numbers like $1, 1, 2, 3, 5, 8, 13, 21, ...$). Do some using the Euclidean algorithm, and then try a bunch with a computer (just type in "$\gcd(a, b)$" into WolframAlpha). Do you notice any patterns? Do you have a conjecture? Can you prove it?